from KALISH & MONTAGUE LOGIC & TECHNIQUES of FORMAL REASONING, HBJ 1964

INFERENCE RULES

(Here ϕ, ψ, χ are to be symbolic formulas and α a variable.)

PRIMITIVE SENTENTIAL RULES:

$$\frac{\begin{array}{c}\phi \rightarrow \psi \\ \phi\end{array}}{\psi}$$ *Modus ponens* (MP)

$$\frac{\begin{array}{c}\phi \rightarrow \psi \\ \sim\psi\end{array}}{\sim\phi}$$ *Modus tollens* (MT)

$$\frac{\phi}{\sim\sim\phi} \qquad \frac{\sim\sim\phi}{\phi}$$ Double negation (DN)

$$\frac{\phi}{\phi}$$ Repetition (R)

$$\frac{\phi \wedge \psi}{\phi} \qquad \frac{\phi \wedge \psi}{\psi}$$ Simplification (S)

$$\frac{\begin{array}{c}\phi \\ \psi\end{array}}{\phi \wedge \psi}$$ Adjunction (Adj)

$$\frac{\phi}{\phi \vee \psi} \qquad \frac{\psi}{\phi \vee \psi}$$ Addition (Add)

$$\frac{\begin{array}{c}\phi \vee \psi \\ \sim\phi\end{array}}{\psi} \qquad \frac{\begin{array}{c}\phi \vee \psi \\ \sim\psi\end{array}}{\phi}$$ *Modus tollendo ponens* (MTP)

$$\frac{\phi \leftrightarrow \psi}{\phi \rightarrow \psi} \qquad \frac{\phi \leftrightarrow \psi}{\psi \rightarrow \phi}$$ Biconditional-conditional (BC)

$$\frac{\begin{array}{c}\phi \rightarrow \psi \\ \psi \rightarrow \phi\end{array}}{\phi \leftrightarrow \psi}$$ Conditional-biconditional (CB)

DERIVED SENTENTIAL RULES:

$$\frac{\begin{array}{c}\phi \rightarrow \psi \\ \sim\phi \rightarrow \psi\end{array}}{\psi} \qquad \frac{\begin{array}{c}\phi \vee \psi \\ \phi \rightarrow \chi \\ \psi \rightarrow \chi\end{array}}{\chi} \qquad \frac{\begin{array}{c}\phi \rightarrow \chi \\ \psi \rightarrow \chi\end{array}}{\phi \vee \psi \rightarrow \chi}$$ Separation of cases (SC)

$$\frac{\sim\phi \rightarrow \psi}{\phi \vee \psi}$$ Conditional-disjunction (CD)

PRIMITIVE QUANTIFICATIONAL RULES:

$$\frac{\bigwedge\alpha\phi}{\psi}\,,$$ Universal istantiation (UI)

$$\frac{\psi}{\bigvee\alpha\phi}\,,$$ Existential generalization (EG)

where ψ comes from ϕ by proper substitution of a term for α;

$$\frac{\bigvee\alpha\phi}{\psi}\,,$$ Existential instantiation (EI)

where ψ comes from ϕ by proper substitution of a variable for α. (See p. 148 for a definition of 'proper substitution'.)

DERIVED QUANTIFICATIONAL RULES:

$$\frac{\sim\bigwedge\alpha\phi}{\bigvee\alpha\sim\phi} \qquad \frac{\bigvee\alpha\sim\phi}{\sim\bigwedge\alpha\phi}$$
$$\frac{\sim\bigvee\alpha\phi}{\bigwedge\alpha\sim\phi} \qquad \frac{\bigwedge\alpha\sim\phi}{\sim\bigvee\alpha\phi}$$ Quantifier negation (QN)